**12.5. A natural monopoly exists in an industry with a demand schedule *P* = 100 - *Q.* The marginal revenue schedule is then *MR* = 100 - 2*Q*. The monopolist operates with a fixed cost *F*, and a total variable cost *TVC* = 20*Q*. The corresponding marginal cost is thus constant and equal to 20.**

**a) Suppose the firm sets a uniform price to maximize profit. What is the largest value of *F* for which the firm could earn zero profit?**

**b) Suppose the firm is able to engage in perfect first degree price discrimination. What is the largest value of *F* for which the firm could earn zero profit?**

a) When the firm sets a uniform price, it sets *MR = MC*: 100 – 2*Q* = 20. The quantity that maximizes profit is therefore *Q* = 40. The profit maximizing uniform price is *P* = 100 – *Q* = 100 – 40 = 60. Profit is *PQ – F -*20*Q* = (60)(40) – *F* – (20)(40) = 1600 – *F*. So the firm could earn at least zero economic profit as long as *F* < 1600.

b) With perfect first degree price discrimination, the firm will charge a price on the demand curve for all units up to the quantity at which the demand curve intersects the marginal cost curve. The demand curve intersects the marginal cost curve when 100 – *Q* = 20, or when *Q* = 80. Total revenue will be the area under the demand curve, or 0.5(100 – 20)80 + 20(80) = 4800. Total cost will be *F* + 20(80) = *F* + 1600. Economic profit will be 4800 – *F* – 1600 = 3200 – *F*. So the firm will be able to earn at least zero economic profit as long as *F* < 3200.

**12.8. Fore! is a seller of golf balls that wants to increase its revenues by offering a quantity discount. For simplicity, assume that the firm sells to only one customer and that the demand for Fore!’s golf balls is *P* = 100 - *Q*. Its marginal cost is *MC* = 10. Suppose that Fore! sells the first block of *Q*1 golf balls at a price of *P*1 per unit.**

**a) Find the profit-maximizing quantity and price per unit for the second block if *Q*1 = 20 and *P*1 = 80.**

**b) Find the profit-maximizing quantity and price per unit for the second block if *Q*1 = 30 and *P*1 = 70.**

**c) Find the profit-maximizing quantity and price per unit for the second block if *Q*1 = 40 and *P*1 = 60.**

**d) Of the three options in parts (a) through (c), which block tariff maximizes Fore!’s total profits?**

a) We can represent the marginal willingness to pay for each unit beyond *Q*1 = 20 as *P =* 100 – (20 + *Q*2) = 80 – *Q*2. The associated marginal revenue is then *MR =* 80 – 2*Q*2, so the profit maximizing second block is *MR = MC*: 80 – 2*Q*2 = 10. Thus *Q*2 = 35 and *P*2 = 80 – 35 = 45. So the firm sells the first 20 units at a price of $80 apiece, while the firm sells any quantity above 20 at $45 apiece. The firm’s total profit will be (80 – 10)\*20 + (45 – 10)\*35 = $2625.

b) The marginal willingness to pay for each unit beyond *Q*1 = 30 is *P =* 70 – *Q*2. So *MR =* 70 – 2*Q*2 and we have *MR = MC*: 70 – 2*Q*2 = 10. Thus *Q*2 = 30 and *P*2 = 40. The firm’s total profit will be (70 – 10)\*30 + (40 – 10)\*30 = $2700.

c) The marginal willingness to pay for each unit beyond *Q*1 = 40 is *P =* 60 – *Q*2. So *MR =* 60 – 2*Q*2 and we have *MR = MC*: 60 – 2*Q*2 = 10. Thus *Q*2 = 25 and *P*2 = 35. The firm’s total profit will be (60 – 10)\*40 + (35 – 10)\*25 = $2625.

d) The option in part (b) yields the highest profits, of $2700.

**12.12. Consider a market with 100 identical individuals, each with the demand schedule for electricity of *P* = 10 - *Q*. They are served by an electric utility that operates with a fixed cost 1,200 and a constant marginal cost of 2. A regulator would like to introduce a two-part tariff, where *S* is a fixed subscription charge and *m* is a usage charge per unit of electricity consumed. How should the regulator set *S* and *m* to maximize the sum of consumer and producer surplus while allowing the firm to earn exactly zero economic profit?**

To maximize the sum of consumer and producer surplus, the regulator must set the usage charge *m* = 2; this will induce consumers to buy units of electricity as long as their willingness to pay is at least as high as the marginal cost of providing electricity service. This means that each consumer will buy 8 units of electricity. There will be zero deadweight loss in the market.

If there were no subscription charge, each consumer would realize a consumer surplus of 0.5\*(10 – 2)\*8 = 32. This means that each consumer will be willing to buy electricity as long as the subscription charge is less than 32. With 100 consumers, the electric utility can then charge each customer a subscription fee of $12 to cover its fixed costs of $1200, leaving each consumer with a consumer surplus of 32 – 12 = 20. So the total revenue for the firm will be the sum of the revenue from the subscription charge (1200) and the revenue from the usage charge 100\*8\*2 = 200. Total revenue will just cover total cost, and the firm will earn zero economic profit.

**12.15. Consider Problem 12.14 with the following change. Suppose the demand for the drug in Europe declines to *QE* = 30 - *PE*. If the firm cannot price discriminate, will it be in the firm’s interest to sell on both continents?**

Let’s start by assuming that the optimal uniform price (*i.e.*, no price discrimination) is one at which the firm would sell in both markets. If the firm cannot price discriminate then 



Inverse demand is *P =* 70 – 0.5*Q*. Setting  implies



At this quantity, price will be . This price exceeds the choke price in Europe, so the firm will not be able to sell any units in Europe. Since the firm will not sell any units in Europe, the firm should set its marginal cost equal to the marginal revenue in the US market: *MRU* = 110 – 2*QU* = *MC =* 10, implying *QU* = 50 and *PU =* 60.

**12.25. A summer theater has a capacity of 200 seats for its Saturday evening concerts. The marginal cost of admitting a spectator is zero up to that capacity. The theater wants to maximize profits and recognizes that there are two kinds of customers. It offers discounts to senior citizens and students, who generally are more price sensitive than other customers. The demand curve for tickets by seniors and students is described by *P*1 = 16 - 0.04*Q*1*,* where *Q*1 is the number of discount tickets sold at a price of *P*1*.* The demand schedule for tickets by customers who do not qualify for a discount is represented by *P*2 = 28 - 0.1*Q*2*,* where *Q*2 is the number of nondiscount tickets sold at a price of *P*2*.* What are the two prices that would maximize profit for the Saturday evening concerts?**

The theater would want to set the prices to equate the marginal revenue for the two types of customers. Thus, it would choose the quantities so that:

*MR1 = 16 – 0.08Q1 = MR2 = 28 – 0.2Q2.*

In addition, the capacity constraint must be satisfied, so that *Q1 + Q2 = 200.*

Thus, *16 – 0.08Q1 = 28 – 0.2(200 – Q1),* which tells us that *Q­1 = 100* and *Q2 = 100.*

Note that this implies that the marginal revenue from each type of ticket is 8. Since this exceeds that marginal cost (which is zero), the firm does want sell all its capacity.

The profit maximizing prices are as follows:

*P1 = 16 – 0.04Q1 = 16 – 0.04(100) = 12. P2 = 28 – 0.1Q2 = 28 – 0.1(100) = 18.*